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EFFECT OF PARTICLES ON THE RATE OF TURBULENT TRANSPORT OF A DUST-LADEN GAS

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Simple approximations in equations of single-point second moments were used to analyze the effect of particles on the intensity of pulsative motion in a nonuniform turbulent flow.

It is well known that even a small concentration of a solid-particle impurity in a turbulent gas flow has a significant effect on the turbulent pulsations of the gas or of the gaseous suspension as a whole [1-7]. Heavy inertialess particles may result in a decrease in turbulence energy due to buoyancy [1, 5]. Light inertialess particles change the turbulence spectrum [2, 5]. The latter particles lead to additional dissipation of the turbulence energy, the magnitude of this dissipation being comparable to the dissipation in a non-dust-bearing gas [3, 6, 7].

The present work examines the turbulent flow of a gas with solid particles which do not interact with one another; in the absence of body forces, the particles interact with the flow only as a result of viscous drag. We obtained closed expressions to correlate the pulsations of particle and gas velocity in the nonuniform flow. Allowing for these expressions, we constructed the system of equations for the second moments of the pulsations of the velocity of the dust-laden gas and used this system as a basis for studying the effect of particle inertia and concentration on the rate of turbulent transport.

1. The following equations of motion of the gas and particles are used to calculate turbulent gas flows with particles:

$$\frac{\partial U_i}{\partial t}+U_k\frac{\partial U_i}{\partial x_k}=-\frac{1}{\rho_1}\frac{\partial P}{\partial x_i}+\nu\frac{\partial^2 U_i}{\partial x_k\partial x_k}-C\frac{\rho_2}{\rho_1}\frac{1}{\tau}(U_i-V_i), \quad (1)$$

$$\frac{\partial V_j}{\partial t}+V_k\frac{\partial V_j}{\partial x_k}=\frac{1}{\tau}(U_j-V_j). \quad (2)$$

Equation (2) can be written in integral form:

$$V_j(x,\ t)=\frac{1}{\tau}\int\limits_0^t ds\exp\left(-\frac{t-s}{\tau}\right)\left\{U_j(x,\ s)-\tau V_k(x,\ s)\frac{\partial V_j(x,\ s)}{\partial x_k}\right\}. \quad (3)$$

We will represent all of the parameters characterizing the motion of the gas and particles in the form of the sum of the mean and pulsative (fluctuation) components. Then for the mean and pulsative components of particle velocity, respectively, we find from (3) that

$$\langle V_j(x,\ t)\rangle=\frac{1}{\tau}\int\limits_0^t ds\exp\left(-\frac{s}{\tau}\right)\Big\{\langle U_j(x,\ t-s)\rangle- -\tau\Big[\langle V_k(x,\ t-s)\rangle\frac{\partial\langle V_j(x,\ t-s)\rangle}{\partial x_k}+\langle v_k(x,\ t-s)\frac{\partial v_j(x,\ t-s)}{\partial x_k}\rangle\Big]\Big\}, \quad (4)$$

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$$v_j(x,\ t)=\frac{1}{\tau}\int\limits_0^t ds\exp\left(-\frac{s}{\tau}\right)\Big\{u_j(x,\ t-s)-$$

$$-\tau\left[\langle V_k(x,\ t-s)\rangle\frac{\partial v_j(x,\ t-s)}{\partial x_k}+v_k(x,\ t-s)\frac{\partial\langle V_j(x,\ t-s)}{\partial x_k}+\right.$$

$$\left.+v_k(x,\ t-s)\frac{\partial v_j(x,\ t-s)}{\partial x_k}-\langle v_k(x,\ t-s)\frac{\partial v_j(x,\ t-s)}{\partial x_k}\rangle\right]\Big\}. \tag{5}$$

Equation (4) includes the effect of longitudinal slip of the particles relative to the gas and their migration across the flow [4, 8, 9].

Multiplying (5) by $u_i(x,\ t)$ and averaging, we write the single-point correlation between the velocity pulsations of the gas and the discrete phase:

$$\langle u_iv_j\rangle=\frac{1}{\tau}\int\limits_0^t ds\exp\left(-\frac{s}{\tau}\right)\Big\{\langle u_i(x,\ t)u_j(x,\ t-s)\rangle-$$

$$-\tau\left[\langle V_k(x,\ t-s)\rangle\langle u_i(x,\ t)\frac{\partial v_j(x,\ t-s)}{\partial x_k}\rangle+\langle u_i(x,\ t)v_j(x,\ t-s)\rangle\times\right.$$

$$\left.\times\frac{\partial\langle V_j(x,\ t-s)\rangle}{\partial x_k}+\langle u_i(x,\ t)\frac{\partial v_j(x,\ t-s)v_k(x,\ t-s)}{\partial x_k}\rangle\right]\Big\}. \tag{6}$$

The nonuniformity of the flow is accounted for in (6) by the term in the brackets.

The correlation of gas velocity pulsation in (6) is calculated from the trajectory of the particles [7]. We will designate

$$\langle u_i(x,\ t)u_j(x,\ t-s)\rangle=B_{ij}\left(x,\ t-\frac{s}{2},\ s\right).$$

The correlation moment B_{ij} (x, t, 0) changes appreciably over time of the order T_0. For simplicity in further calculations, we assign B_{ij} (x, t, s) in the form of the step function

$$B_{ij}(x,\ t,\ s)=\begin{cases}0 & \text{at}\quad s>T,\\ B_{ij}(x,\ t,\ 0) & \text{at}\quad 0\leqslant s\leqslant T,\end{cases}$$

where T is the time of interaction of a particle with a turbulent mole. Henceforth, we will examine the flow without allowance for the mean interphase slip, which is valid when $T_0>\tau$ ($T_0\sim L/\langle U\rangle$). We will also ignore pulsative slip. In this case, the time of interaction of the particle with the turbulent mole T coincides with the lifetime of the mole (the turbulence time scale) [7].

The main property of the correlation function over small time intervals [10] is satisfied for the chosen approximation $B_{ij}(x,\ t,\ s)$:

$$\left.\frac{\partial B_{ij}(x,\ t,\ s)}{\partial s}\right|_{s=0}=0.$$

Together with the condition $\tau/T_0<1$, we adopt the condition $T/T_0\ll 1$. We limit ourselves in the expansion of B_{ij} in the second argument to terms of the order τ/T_0. Then, integrating in (6) over the region $|t-s|\leqslant T$, we obtain a simple closed expression for correlation of the velocities of the gas and particles through the correlation moments of the gas

$$\langle u_iv_j\rangle=(1-f_1)\langle u_iu_j\rangle-T\Omega f_2\left[\frac{1}{2}\frac{\partial\langle u_iu_j\rangle}{\partial t}+\right.$$

$$\left.+\frac{\langle V_k\rangle}{2}\frac{\partial\langle u_iu_j\rangle}{\partial x_k}+\frac{(1-f_1)}{2}\frac{\partial\langle u_iu_ju_k\rangle}{\partial x_k}+\frac{\partial\langle V_j\rangle}{\partial x_k}\langle u_iu_k\rangle\right], \tag{7}$$

where $\Omega = \tau/T$ is a parameter characterizing the inertia of the particles, and

$$f_1 = \exp(-1/\Omega),\ f_2 = 1 - (1/\Omega + 1)\exp(-1/\Omega). \tag{8}$$

We similarly calculate the correlation between the pulsations of the velocity components of the discrete phase:

$$\langle v_i v_j \rangle = (1 - f_1)\langle u_i u_j \rangle - T\Omega f_2 \left[\frac{\partial \langle u_i u_j \rangle}{\partial t} + \langle V_k \rangle \frac{\partial \langle u_i u_j \rangle}{\partial x_k} + \right. \\ \left. + (1 - f_1)\frac{\partial \langle u_i u_j u_k \rangle}{\partial x_k} + \frac{\partial \langle V_i \rangle}{\partial x_k}\langle u_j u_k \rangle + \frac{\partial \langle V_j \rangle}{\partial x_k}\langle u_i u_k \rangle \right]. \tag{9}$$

The terms in the brackets in (7), (9) are of the same order of magnitude as $\langle u_i u_j \rangle / T_0$. It follows from (7), (9) for inertialess particles ($\Omega \to 0$) that $\langle v_i v_j \rangle = \langle u_i v_j \rangle \to \langle u_i u_j \rangle + O(\Omega)$. For inertial particles ($\Omega \to \infty$), we have $\langle v_i v_j \rangle = \langle u_i v_j \rangle \to 0$, i.e., the particles are not brought into pulsative motion. The behavior $\langle v_i v_j \rangle$ and $\langle u_i v_j \rangle$ at high and low Ω is in qualitative agreement with the data of other authors e.g., [4. 7, 11]. Thus, Eqs. (7), (9) are considered valid throughout the range of variation in the parameter Ω.

Without consideration of the effect of particle acceleration, the equation below for the mean velocity of the dust-laden flow follows from Eqs. (1), (4)

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} + \langle \Phi \rangle \langle V_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k} = -\frac{1}{\rho_1}\frac{\partial \langle P \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle U_i \rangle}{\partial x_k \partial x_k} - (1 + f_2 \langle \Phi \rangle)\frac{\partial \langle u_i u_k \rangle}{\partial x_k}.$$

2. The equations for the second moments of the velocity pulsations of the dust-laden gas coincide with the corresponding equations for a pure gas [12], except for the additional term due to the interphase interaction:

$$\varepsilon_{ij} = -\frac{\langle \Phi \rangle}{\Omega T}[\langle u_i(u_j - v_j)\rangle + \langle u_j(u_i - v_i)\rangle].$$

The change in turbulence energy due to the imparting of pulsative motion to the particles is considered in the expression for ε_{ij}.

Using the relation for the correlations of the velocity pulsations of the gas and discrete phase (7), we write the system of balance equations for the second moments of the velocity pulsations of the gas carrying the particles:

$$(1 + \langle \Phi \rangle f_2)\frac{\partial \langle u_i u_j \rangle}{\partial t} + (\langle U_k \rangle + \langle \Phi \rangle f_2 \langle V_k \rangle)\frac{\partial \langle u_i u_j \rangle}{\partial x_k} = \\ = -[1 + \langle \Phi \rangle f_2 (1 - f_1)]\frac{\partial \langle u_i u_j u_k \rangle}{\partial x_k} - \left(\frac{\partial \langle U_i \rangle}{\partial x_k} + \langle \Phi \rangle f_2 \frac{\partial \langle V_i \rangle}{\partial x_k}\right) \times \\ \times \langle u_j u_k \rangle - \left(\frac{\partial \langle U_j \rangle}{\partial x_k} \cdot \langle \Phi \rangle f_2 \frac{\partial \langle V_j \rangle}{\partial x_k}\right)\langle u_i u_k \rangle - \\ - \frac{1}{\rho_1}\left(\frac{\partial \langle u_i p \rangle}{\partial x_k} + \frac{\partial \langle u_j p \rangle}{\partial x_k}\right) + \left\langle \frac{p}{\rho_1}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\rangle + \\ + \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial x_k \partial x_k} - 2\nu \left\langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}\right\rangle - 2\langle \Phi \rangle \frac{f_3}{T}\langle u_i u_j \rangle, \tag{10}$$

where

$$f_3 = f_1/\Omega = \exp(-1/\Omega)/\Omega. \tag{11}$$

It can be seen from (10) that the presence of particles in the gas should be considered in terms describing convection, turbulent diffusion, and the generation of pulsative energy from the mean motion. There also appears a term connected with additional dissipation due to the incomplete entrainment of the particles by the gas (the last term in the equation).

For intertialess particles ($\Omega \ll 1$, $f_2 \to 1$, $f_3 \to 0$), the additional dissipation is zero, the particles are almost completely brought into pulsative motion by the gas, and Eq. (10) becomes the equation for the second moments of the pulsation of a pure gas with density $\rho_1 (1 + \langle\Phi\rangle)$. Inertial particles ($\Omega \gg 1$, $f_3 \to 1/\Omega$, $f_2 \to 0$) weaken the turbulent pulsations of the gas.

Figure 1 shows the functions f_2 and f_3, which are proportional to the additional generation of turbulence energy from the mean motion and additional dissipation. The function f_3 in the region $\Omega > 1$ is in accord with the expressions used in [3, 6, 7] for dissipation in flow about the particles.

3. Let us analyze the effect of the particles on the rate of turbulent transport at high turbulent Reynolds numbers $Re_E = E^{1/2}l/\nu$. To describe the dissipative and volume terms we will use Rotta's approximation [13]:

$$\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right\rangle = \frac{b}{3} \frac{E^{3/2}}{l} \delta_{ij},$$

$$-\left\langle \frac{p}{\rho_1} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right\rangle = k \frac{E^{1/2}}{l} \left(\langle u_i u_j \rangle - \frac{2}{3} E \delta_{ij} \right).$$

Ignoring the convective and diffusional terms in Eq. (10), we write the following system for nontrivial components of the second moments:

$$\begin{gathered}
(1 + \langle \Phi \rangle f_2) \frac{\partial \langle U \rangle}{\partial y} \langle u_x u_y \rangle + \frac{k}{2} \frac{E^{1/2}}{l} \left(\langle u_x^2 \rangle - \frac{2}{3} E \right) + \frac{b}{3} \frac{E^{3/2}}{l} + \langle \Phi \rangle f_3 \frac{\langle u_x^2 \rangle}{T} = 0, \\
\frac{k}{2} \frac{E^{1/2}}{l} \left(\langle u_y^2 \rangle - \frac{2}{3} E \right) + \frac{b}{3} \frac{E^{3/2}}{l} + \langle \Phi \rangle f_3 \frac{\langle u_y^2 \rangle}{T} = 0, \\
\frac{k}{2} \frac{E^{1/2}}{l} \left(\langle u_z^2 \rangle - \frac{2}{3} E \right) + \frac{b}{3} \frac{E^{3/2}}{l} + \langle \Phi \rangle f_3 \frac{\langle u_z^2 \rangle}{T} = 0, \\
\frac{1}{2} (1 + \langle \Phi \rangle f_2) \frac{\partial \langle U \rangle}{\partial y} \langle u_y^2 \rangle + \frac{k}{2} \frac{E^{1/2}}{l} \langle u_x u_y \rangle + \langle \Phi \rangle f_3 \frac{\langle u_x u_y \rangle}{T} = 0.
\end{gathered} \tag{12}$$

In writing system (12), we assume that $\partial\langle U\rangle/\partial y \gg \partial\langle U\rangle/\partial x$. For a known distribution of mean velocity and turbulence scale, we can use system (12) to obtain formulas for the turbulence energy and the turbulent shear stress of a gas with particles:

$$E = \frac{2}{3} \frac{k-b}{k^2 b} \frac{(1 + \langle \Phi \rangle f_2)^2}{\left(1 + \frac{2 \langle \Phi \rangle f_3 l}{kTE^{1/2}}\right)^2 \left(1 + \frac{2 \langle \Phi \rangle f_3 l}{bTE^{1/2}}\right)} \left(l \frac{\partial \langle U \rangle}{\partial y} \right)^2 = \frac{2}{3} \frac{k-b}{k^2} \Psi_E \left(l \frac{\partial \langle U \rangle}{\partial y} \right)^2, \tag{13}$$

$$\begin{gathered}
\langle u_x u_y \rangle = -\left[\frac{2(k-b)}{3k^2 b} \right]^{3/2} \frac{(1 + \langle \Phi \rangle f_2)^2}{\left(1 + \frac{2 \langle \Phi \rangle f_3 l}{kTE^{1/2}}\right)^2 \left(1 + \frac{2 \langle \Phi \rangle f_3 l}{bTE}\right)^{1/2}} \times \\
\times \left(l \frac{\partial \langle U \rangle}{\partial y} \right)^2 = -\left[\frac{2(k-b)}{3k^2} \right]^{3/2} \Psi_\tau \left(l \frac{\partial \langle U \rangle}{\partial y} \right)^2.
\end{gathered} \tag{14}$$

Equations (13), (14) are a generalization of Prandtl's laws for turbulence energy and shear stress in the case of a dust-bearing gas. The coefficients Ψ_E, Ψ_τ characterize the degree of the effect of the particles on the rate of turbulent transport.

Figure 2 shows the dependence of the coefficients Ψ_E and Ψ_τ on the parameter $\langle\Phi\rangle$ ($\langle\Phi\rangle$ is the mean ratio of the mass of the particles to the mass of the gas). The lifetime of a turbulent mole T, entering into the parameter of particle inertia Ω, is expressed through the turbulence energy and the three-dimensional turbulence scale by means of the relation $T = l/E^{1/2}$. The values of the constants are chosen from a comparison with experiments for the characteristics of the boundary turbulence of a non-dust-bearing gas: $b/k = 0.125$ and $k = 1.12$. It is apparent that the greatest weakening of the turbulent pulsations is seen at

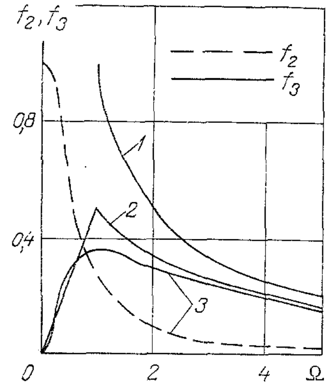

Fig. 1

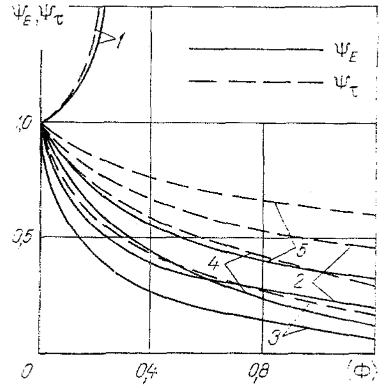

Fig. 2

Fig. 1. Functions f_2 and f_3 and approximations of other authors for the additional dissipation: 1) [6, 7]; 2) [3]; 3) from Eqs. (8), (11).

Fig. 2. Effect of the particles on the rate of turbulent transport: 1) $\Omega = 0.1$; 2) 0.5; 3) 1.0; 4) 5.0; 5) 10.0.

$\Omega \sim 1$, which is connected with the maximum of the additional dissipation caused by the interphase pulsative (fluctuation) velocity. At low values of Ω, as a result of entrainment of particles by the gas, there is an increase in the rate of turbulent transport.

Thus, we have obtained a system of equations for the moments of the pulsations of velocity of a gas which allows us to consider the change in the rate of turbulent transport of the dust-laden flow throughout the range of variation in particle inertia.

NOTATION

x_i, axes of cartesian coordinate system ($x_1 = x$, direction along the flow; $x_2 = y$, direction across the flow; $x_3 = z$); U_i, $\langle U_i \rangle$, u_i, components of the total, mean, and fluctuation velocities of the gas; V_i, $\langle V_i \rangle$, v_i, components of the total, mean, and fluctuation velocities of the particles; P, ⟨P⟩, p, total and mean pressure and pressure pulsation; t, time; ρ_2, density of the material of the particles; ρ_1, density of the pure gas; $\tau = (2/9)(\rho_2/\rho_1)(a^2/\nu)$, relaxation time of a spherical particle of radius a; ν, kinematic viscosity of the pure gas; C, ⟨C⟩, c, total and mean volumetric concentration of the particles and pulsation of the volumetric concentration; γ, coefficient of Brownian diffusion of the particles; T, time scale of the turbulence; $E = 1/2\ [\langle u_x^2 \rangle + \langle u_y^2 \rangle + \langle u_z^2 \rangle]$, turbulence energy of a unit mass of the gas; l, three-dimensional turbulence scale; ⟨U⟩, mean flow velocity along the x axis; $\langle \Phi \rangle = \rho_2/\rho_1 \langle C \rangle$; L, characteristic dimension of the channel.

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LAMINAR-TO-TURBULENT FLOW TRANSITION UNDER THE ACTION OF ACOUSTIC OSCILLATIONS

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UDC 532.517

The formation of two conical flows in the transition from laminar flow to the turbulent part of a free jet under the action of acoustic oscillations is observed experimentally.

The process of transition from laminar to turbulent flow under the action of acoustic oscillations is of considerable scientific and practical interest. In particular, the results of investigations are used in the design of pneumoacoustic devices [1]. Flow-visualization techniques afford one of the methods of studying the physical patterns of aerodynamic processes.

We have used smoke particles to visualize a free laminar jet on the experimental arrangement shown in Fig. 1.

The smoke generator saturated with smoke particles the flow entering the capillary tube 2, at whose exit a visible jet was formed. The supply pressure was chosen so that the jet at the exit from the capillary tube would have a laminar zone with a length of roughly 20d. The parameters of the flow were as follows in this case: Re $\approx$ 2000, d = 0.6 mm. The jet was irradiated with an acoustic signal generated by the piezoelectric crystal 4, to the faces of which was applied an alternating voltage from the oscillator 5. The visual flow was photo-

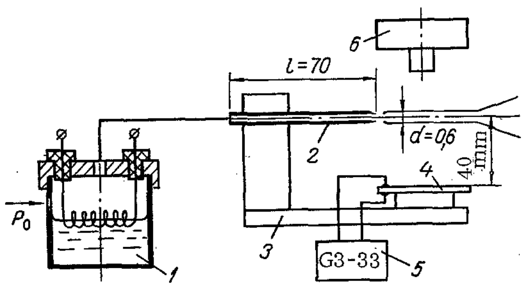

Fig. 1. Diagram of the experimental arrangement for visualizing a free laminar jet: 1) oil-burning smoke generator; 2) capillary tube; 3) bracket; 4) piezoelectric crystal; 5) audio oscillator; 6) camera attachment.

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